



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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4037/12

May/June 2024

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

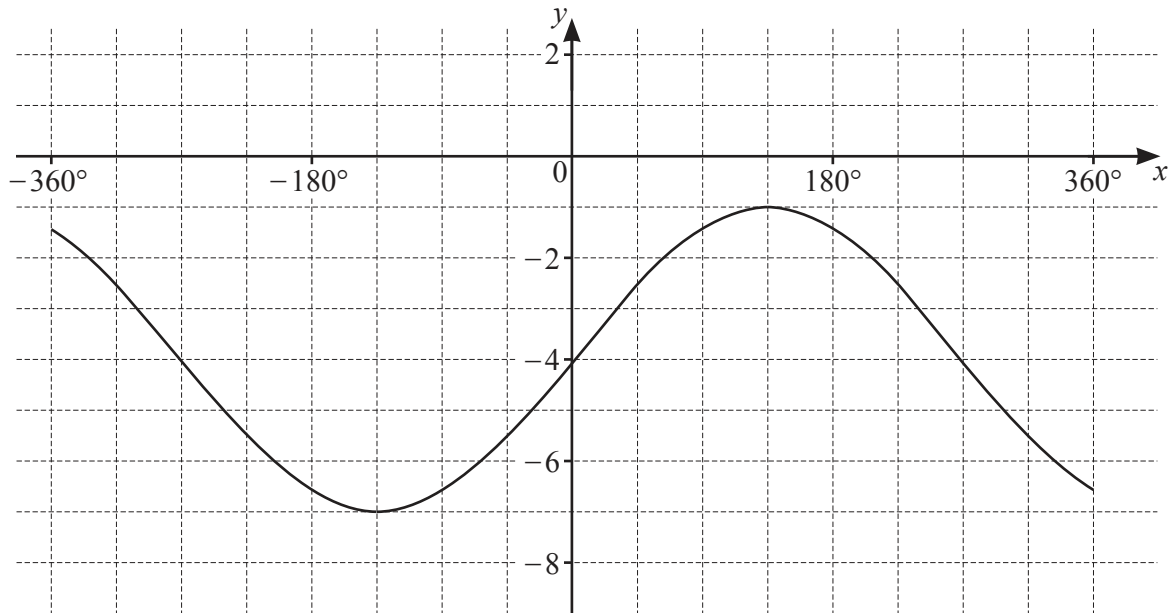
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows the graph of $y = a \sin bx + c$ for $-360^\circ \leq x \leq 360^\circ$, where a , b and c are constants. Find the values of a , b and c . [3]

- 2 Given that $\log_3 r + 2 \log_9 s = 8$, find the value of rs . [3]

- 3 Given that $y = \tan \frac{x}{2}$, find the exact value of $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$. [4]

4 A team of 8 people is to be formed from 6 teachers, 5 doctors and 4 police officers.

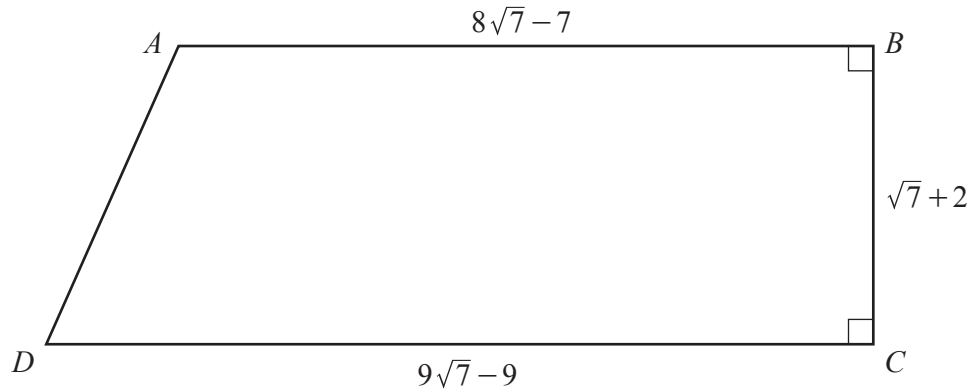
(a) Find the number of teams that can be formed. [1]

(b) Find the number of teams that can be formed without any teachers. [1]

(c) Find the number of teams that can be formed with the same number of doctors as teachers. [4]

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.



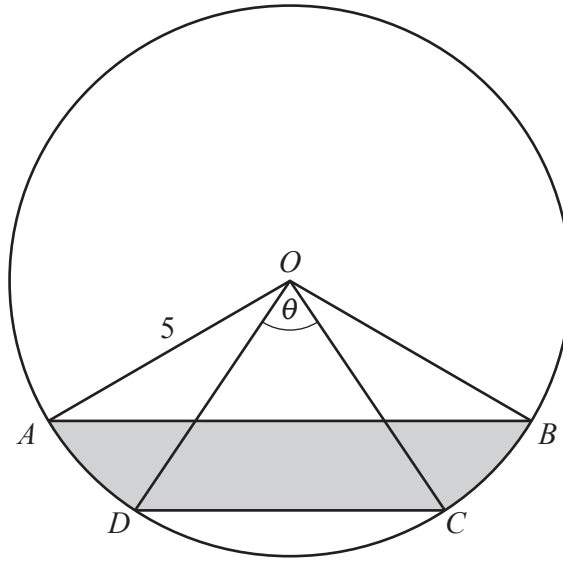
The diagram shows the trapezium $ABCD$. The lengths of AB , BC and CD are $8\sqrt{7} - 7$, $\sqrt{7} + 2$ and $9\sqrt{7} - 9$ respectively. The line BC is perpendicular to the lines AB and CD .

- (a) Find the perimeter of the trapezium, giving your answer in its simplest form. [3]

- (b) Find the area of the trapezium, giving your answer in the form $p\sqrt{7} + q$, where p and q are rational numbers. [3]

- (c) Find $\cot DBC$, giving your answer in the form $r\sqrt{7} + s$, where r and s are simplified rational numbers. [3]

- 6 In this question, all lengths are in metres and all angles are in radians.



The diagram shows a circle with centre O and radius 5. The points A , B , C and D lie on the circumference of the circle. Angle $DOC = \theta$. Angle $AOD = \text{angle } COB = 0.5$. The length of the minor arc DC is 3.75.

- (a) Show that $\theta = 0.75$. [1]

- (b) Find the perimeter of the shaded region. [5]

(c) Find the area of the shaded region.

[3]

- 7 (a) The line $y = 3x - 2$ intersects the curve $2x^2 - xy + y^2 = 2$ at the points A and B . The point C with coordinates $\left(k, \frac{7}{8}\right)$ lies on the perpendicular bisector of the line AB . Find the exact value of k . [9]

- (b) The point D lies on the perpendicular bisector of AB such that D is a reflection of C in the line AB . Find the coordinates of D . [2]

8 A curve has equation $y = \frac{(3x^2 - 5)^{\frac{1}{3}}}{x + 4}$.

(a) Show that $\frac{dy}{dx}$ can be written in the form $\frac{Ax^2 + Bx + C}{(3x^2 - 5)^{\frac{2}{3}}(x + 4)^2}$, where A , B and C are integers. [5]

(b) Hence find the x -coordinates of the stationary points on the curve. Give your answers in their simplest exact form. [3]

9 In this question, all distances are in metres and time, t , is in seconds.

A particle P moves with a speed of 14.5 parallel to the vector $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$.

(a) Find the velocity vector of P . [2]

Initially, P has position vector $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

(b) Write down the position vector of P at time t . [2]

A second particle Q has position vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix}t$ at time t .

(c) Find, in terms of t , the distance between P and Q at time t . Simplify your answer. [4]

(d) Hence show that P and Q never collide. [2]

10 (a) The first 3 terms of an arithmetic progression are $3 \sin 2x$, $5 \sin 2x$, $7 \sin 2x$.

(i) Show that the sum to n terms of this arithmetic progression can be written in the form $n(n+a) \sin 2x$, where a is a constant. [3]

(ii) Given that $x = \frac{2\pi}{3}$, find the exact sum of the first 20 terms. [2]

(b) The first 3 terms of a geometric progression are $\ln 2y$, $\ln 4y^2$, $\ln 16y^4$.

(i) Find the n th term of this geometric progression. [2]

(ii) Find the sum to n terms of this geometric progression, giving your answer in its simplest form. [2]

(c) The first 3 terms of a different geometric progression are $\left(2w - \frac{1}{4}\right)$, $\left(2w - \frac{1}{4}\right)^2$, $\left(2w - \frac{1}{4}\right)^3$.

Find the values of w for which this geometric progression has a sum to infinity. [3]

Question 11 is printed on the next page.

11 (a) Given that $y = x^2 \ln x$, find $\frac{dy}{dx}$. [2]

(b) Hence find $\int x \ln x \, dx$. [3]

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